Name:

Math 10a
October 2, 2014
Quiz #4

1. Let $f(x) = \sin(x)$. What is the second order Taylor polynomial for f centered at $x = \frac{\pi}{2}$?

$$1 - \frac{1}{2}\left(x - \frac{\pi}{2}\right)^2.$$

2. Let $f(x) = \ln(x)$. What is the third order Taylor polynomial for f centered at $x = 1$?

$$x - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{6}$$

3. Estimate $\ln(1.1)$ using its third order Taylor polynomial centered at $x = 1$.

$$\ln(1.1) \approx 1.1 - \frac{(1.1-1)^2}{2} + \frac{(1.1-1)^3}{6}.$$

4. In summation notation, write down the nth order Taylor polynomial to e^x centered at $x = 0$.

$$\sum_{k=0}^{n} \frac{x^k}{k!}.$$

5. (a) The equation $x^3 + 2x + 2 = 0$ has only one real solution. Why?

   $f(x) = x^3+2x+2$ has a positive derivative $(3x^2+2)$ and so f is always increasing. The solutions to $x^3 + 2x + 2$ correspond to where f crosses the x-axis and, since it is always increasing, it can only cross once.

   (b) It looks to me like the solution should be pretty close to -1, since $(-1)^3 - 2 + 2 = -1$ isn't terribly big. Improve on this estimate with *two* iterations of Newton's method. Express your answer as a fraction in simplest terms.

$$\begin{aligned} x_1 &= -1 \\ x_2 &= -1 - \frac{(-1)^3 + 2(-1) + 2}{3(-1)^2 + 2} = -\frac{4}{5} \\ x_3 &= -\frac{4}{5} - \frac{\left(-\frac{4}{5}\right)^3 + 2\left(-\frac{4}{5}\right) + 2}{3\left(-\frac{4}{5}\right)^2 + 2} = -\frac{27}{35}. \end{aligned}$$

6. Compute the infinite sum

$$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \cdots .$$

$$= \frac{1}{1 - \frac{1}{3}}.$$